Vector Calculus, Linear Algebra, and Differential Forms: A Unified Approach

Errata for the Second Edition

Updated March 8, 2005

Inside front cover

inf: infimum, not minimum

Chapter 0

Page 12 Example 0.4.4: The first sentence after the displayed equation should be "This can be evaluated only if $x^2 - 3x + 2 \geq 0$, which happens if $x \leq 1$ or $x \geq 2$." The second sentence should be "So the natural domain is $(-\infty, 1] \cup [2, \infty)$."

Page 15 Example 0.4.10: The range of f should be the real numbers, not the real positive numbers.

Page 18 The definition of the truncation $[x]_k$ and the discussion in the following paragraph of one number being larger than another fails to take into account the non-fractional part. For a corrected version, please see the complete list of comments and errata, or the errata for Chapter 0.

Page 19 The proof of Theorem 0.5.3 also fails to take into account the non-fractional part. The second paragraph of the proof has been rewritten; please see the complete list of comments and errata, or the errata for Chapter 0.

Page 20 Last line of the proof of Theorem 0.5.7: $A - a_n \leq A - a_N$, not $A - a_n < A - a_N$.

Page 21 Theorem 0.5.8, first line of proof: the summand should be in parentheses: $\sum_{n=1}^{\infty}(a_n + |a_n|)$.

Page 23 Two lines before Equation 0.6.4: "of degree <= 2 with $|ai|$ <= 2", not " … with ai <= 2".

Chapter 1

Page 48 Figure 1.2.5: On the right, the final matrix should be $A(BC)$, not $(AB)C$.

Page 61 In the first paragraph of the remark, the mention of "feedback" is incorrect. Feedback is compatible with linearity.

Page 65 In Theorem 1.3.11, we assumed that the composition $T \circ S$ is a linear transformation. We should have stated this as part of the theorem and then proved it. For details, please see the complete list of comments and errata.

Page 76 The proof of Theorem 1.4.5 should start with the sentence "If either $\vec{\mathbf{v}}$ or $\vec{\mathbf{w}}$ is $\mathbf{0}$, the statement is obvious, so suppose both are nonzero."

Page 85 Figure 1.4.12: The arc is misplaced. It should go from $\mathbf{h}$ to $\mathbf{a}$.

Page 93 In Equation 1.5.13, U should be $\overline{U}$.

Page 101 Proof of Theorem 1.5.23: since $\mathbf{x}_0$ is not guaranteed to be in U, we should replace $\mathbf{f}(\mathbf{x}_0)$ by $\mathbf{a} := \lim_{\mathbf{x}\to\mathbf{x}_0} \mathbf{f}(\mathbf{x})$ and $\mathbf{g}(\mathbf{x}_0)$ by $\mathbf{b} := \lim_{\mathbf{x}\to\mathbf{x}_0} \mathbf{g}(\mathbf{x})$.

Page 101 The notation in the proof of Theorem 1.5.24 is a little mixed up. The theorem and proof should read

Theorem 1.5.24 (Limit of a composition). *Let $U \subset \mathbb{R}^n$, $V \subset \mathbb{R}^m$ be subsets, and $\mathbf{f} : U \to V$ and $\mathbf{g} : V \to \mathbb{R}^k$ be mappings, so that $\mathbf{g} \circ \mathbf{f}$ is defined in U. If $\mathbf{y}_0 := \lim_{\mathbf{x}\to\mathbf{x}_0} \mathbf{f}(\mathbf{x})$ and $\mathbf{z}_0 := \lim_{\mathbf{y}\to\mathbf{y}_0} \mathbf{g}(\mathbf{y})$ both exist, then $\lim_{\mathbf{x}\to\mathbf{x}_0} \mathbf{g} \circ \mathbf{f}(\mathbf{x})$, exists, and*

$$\lim_{\mathbf{x}\to\mathbf{x}_0} (\mathbf{g} \circ \mathbf{f})(\mathbf{x}) = \mathbf{z}_0. \tag{1.5.45}$$

Proof. For all $\epsilon > 0$ there exists $\delta_1 > 0$ such that if $|\mathbf{y} - \mathbf{y}_0| < \delta_1$, then $|\mathbf{g}(\mathbf{y}) - \mathbf{z}_0| < \epsilon$. Next, there exists $\delta > 0$ such that if $|\mathbf{x} - \mathbf{x}_0| < \delta$, then $|\mathbf{f}(\mathbf{x}) - \mathbf{y}_0| < \delta_1$. Hence

$$|\mathbf{g}\big(\mathbf{f}(\mathbf{x})\big) - \mathbf{z}_0| < \epsilon \quad \text{when} \quad |\mathbf{x} - \mathbf{x}_0| < \delta. \quad \square \tag{1.5.46}$$

Page 102 Line 3: "the limit does not exist", not "the limit may not exist".

Page 103 Margin note, end of first paragraph: "for different values of $\mathbf{x}_0$", not "for different values of $\mathbf{x}$".
Theorem 1.5.28 (e): "even if $\mathbf{f}$ is not continuous at $\mathbf{x}_0$", not "even if $\mathbf{f}$ is not defined at $\mathbf{x}_0$".

Page 105 The proof of Proposition 1.5.34 should read

Set $\vec{\mathbf{a}}_i = \begin{bmatrix} a_{1,i} \\ \vdots \\ a_{n,i} \end{bmatrix}$. Then $|a_{k,i}| \le |\vec{\mathbf{a}}_i|$, so $\sum_{i=1}^{\infty} |a_{k,i}|$ converges, so by Theorem 0.5.8, $\sum_{i=1}^{\infty} a_{k,i}$ converges. Proposition 1.5.34 then follows from Proposition 1.5.13.

Page 106 Third line of proof of Proposition 1.5.35: $S_k(I - A) = I - A^{k+1}$, not $S_l(I - A) = I - A^{k+1}$.

Page 108 Exercise 1.5.12: We must assume $u(\epsilon) > 0$.

Page 109 Exercise 1.5.20, part (a): n was used with two different meanings.

Page 113 Six lines from the bottom, "the digits $0, 1, 1, 2, 4$" should be "the digits $0, 1, 2, 3, 4$"

Page 123 4th line from bottom: "of degree 1 or 2", not "of degree 1."

Page 125 Hint for Exercise 1.6.7: "minimum" not "maximum."

Page 126 It is incorrect to ascribe the motions of a pendulum to feedback.

Page 133 Equation 1.7.19: the 0 in $\lim \vec{\mathbf{h}} \to 0$ should be bold, since it is a vector.

Page 137 Equation 1.7.37: everything after $\lim_{h\to 0}$ should be in parentheses.

Page 138 Remark: In several places we wrote $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ when we meant $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$. The $\vec{\mathbf{v}}$ in the expression $\left[\mathbf{D}f \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right] \vec{\mathbf{v}}$ does not belong there. The last half of the remark should read:

... to a step of length $\sqrt{5}$ in the direction $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$. To take a step of length 1 in that direction, starting at the origin, we would multiply $\left[\mathbf{D}f \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right]$ by $\begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$, which has length 1, to get a rate of ascent (at time 0) of $19/\sqrt{5} \approx 8.5$. In which direction is the function increasing faster, $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ or $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$?

In the footnote, $36/5 \approx 7.2$ should be $36/5 = 7.2$.

Page 149 Long displayed equation after Equation 1.8.15, line 2: in the denominator at far right, $(f(\mathbf{a}))^2$ should be $(f(\mathbf{a}))$.

Page 151 Equation 1.8.22: in the second matrix on the right side, the second entry in the third row should be 2, not 1.

Page 154 Exercise 1.8.10: $\mathbf{f} : \mathbb{R}^2 \to \mathbb{R}^2$ should be $f : \mathbb{R}^2 \to \mathbb{R}$.

Page 157 Equation 1.9.15, left side: everything after $\lim_{x\to 0}$ should be in parentheses.

Page 161 The $=$ in the first line of Equation 1.9.25 should be $\leq$.

Chapter 2

Page 178 Margin note: The vector $\vec{\mathbf{b}}$ does not contain the solutions.

Page 185 Exercise 2.2.6, part (a): "have a solution" (not "have a unique solution").

Page 188 Part (3) of Definition 2.3.6: "$i \neq j$", not $1 \neq j$.

Page 206 Part (a) of Exercise 2.4.13 was poorly stated. It should be:

(a) For $n = 1, n = 2, n = 3$, write the system of linear equations which the $a_{0,n}, \dots, a_{n,n}$ must satisfy so that the integral of 1 is exact, the integral of x is exact, and so on, until you get to x^n.

Exercise 2.5.14, part (c): W should be W_t.

Page 214 In the second box, statement 6 is incorrect. It should be: The row-reduced matrix $\widetilde{A}$ has no nonpivotal column.

Page 224 Part (c) of Exercise 2.5.20: "For any vectors $\vec{\mathbf{b}} \in \mathbb{R}^n$", not "for any numbers $\vec{\mathbf{b}} \in \mathbb{R}^n$".

Page 231 Equation 2.6.18: $+\cdots+$, not $+\cdots$.

Page 236 Exercise 2.6.3: The four matrices do not form a basis, since $\underline{\mathbf{v}}_3 = -\underline{\mathbf{v}}_4$.

Exercise 2.6.5: After the displayed equation, $\Phi^{-1}_{\{\underline{\mathbf{v}}\}}$ should be $\Phi_{\{\underline{\mathbf{v}}\}}$.

Page 237 Exercise 2.6.11: A_a should be A.

Page 243 In Definition 2.7.4 we should have required that $U \subset \mathbb{R}^n$ be open.

Page 246 Example 2.7.11 uses a different order for the subscripts of c than that given in Proposition 2.7.10.

Page 249 Statement of Theorem 2.7.13: in the next-to-last line, it should be "has a unique solution in the closed ball $\overline{U_0}$". To see why this is necessary, consider Example 2.8.1, where Newton's method converges to 1, which is not in U_0 but is in its closure.

Page 250 At the end of Equation 2.7.55 we should write < 1.2, not < 2.

Equation 2.7.56 contains several errors. In the second rows of the matrices on the right, two minus signs should be pluses. In the third line of Equation 2.7.56, the first $=$ in the last line should be $\leq$, the 4 under the square root should be 8, and the 2 after the second $=$ should be $2\sqrt{2}$.

Page 251 In the first line, $M = 2\sqrt{2}$, not $M = 2$.

Equation 2.7.57 should be:

$$|\vec{F}(\mathbf{a}_0)| \left|[\mathbf{D}\vec{F}(\mathbf{a}_0)]^{-1}\right|^2 M \leq .1 \cdot 1.2 \cdot 2\sqrt{2} \approx .34 < .5. \qquad 2.7.57$$

Page 253 As on page 246, the order of subscripts for c is wrong in three places at the bottom of the page. Below, the starred entries have been corrected:

$$\begin{array}{ll} \sup|D_1D_1f_1| \le 3 = c_{1,1,1} & *\sup|D_1D_1f_2| = 0 = c_{2,1,1} \\ \sup|D_1D_2f_1| \le 1 = c_{1,2,1} & *\sup|D_1D_2f_2| = 0 = c_{2,2,1} \\ *\sup|D_2D_2f_1| \le 1 = c_{1,2,2} & \sup|D_2D_2f_2| = 2 = c_{2,2,2}. \end{array}$$

Page 260 There should be no vector $\vec{\mathbf{v}}$ in Definition 2.8.6; "when $|\vec{\mathbf{x}}| = 1$" should be when "$\mathbf{x} \in \mathbb{R}^n$ and $|\vec{\mathbf{x}}| = 1$".

Page 264 Exercise 2.8.8: In the displayed equation, D should be D^2:

$$\|A\| = \left(\frac{|A|^2 + \sqrt{|A|^4 - 4D^2}}{2} \right)^{1/2}.$$

Page 270 We never proved Equation 2.9.13! Moreover, it is wrong, which shows how dangerous it is to omit proofs. See the complete list of comments and errata for the correct equation and proof.

Page 271 At the end of the first paragraph after Figure 2.9.6: "look at condition (3a)" should be "look at condition (1)." In the next paragraph, "condition (3b)e" should be "condition (2)".

Page 273 First line after Equation 2.9.18: Equation 2.9.11, not Equation 2.9.24.

Page 280 The second margin note is false. Using the second partial derivative method in Example 2.9.15 is perfectly possible and gives a Lipschitz ratio of $2\sqrt{3}$.

Page 281 Second line: "diagonal matrices" should be "diagonal entries."

Page 284 In Execise 2.9.4, the matrix $\begin{bmatrix} -3 & 0 \\ 0 & 3 \end{bmatrix}$ should be $\begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$. This matrix appears three times.

Page 290 Exercise 2.33: in two places, "of degree" should be "of degree at most":

"q_1 and q_2 are polynomials of degrees at most $k_2 - 1$ and $k_1 - 1$" and "the space of polynomials of degree at most $k_1 + k_2 - 1$."

Chapter 3

Page 294 Caption to Figure 3.1.3: we wrote that "the curve in $I_1 \times J_1$ can also be thought of as the graph of a function expressing $x \in I_1$ as a function of $y \in J_1$", but this is wrong, because we made J_1 too big; there are values of $y \in J_1$ that give no values in $I - 1$.

Page 300 Margin notes: in the second line of the first note, $[\mathbf{D}F(\mathbf{a})]$should be $[\mathbf{DF}(\mathbf{a})]$. In the next note, $\mathbf{F}$ should be F. The last note should read, "More generally, for an $(n-1)$-dimensional manifold in any $\mathbb{R}^n$, ... "

Page 307 Five lines from the bottom: "$[\mathbf{D}\gamma(\mathbf{u})]$ be one to one", not "$[\mathbf{D}\gamma(\mathbf{u})]$ be one".

Exercise 3.1.26: we used A both to denote the $A(n, n)$ (the space of antisymmetric $n \times n$ matrices) and to denote the matrice A. The matrix A is $n \times n$.

Page 316 Exercise 3.1.28, part (c): $\mathbf{g}$, not g.

Definition 3.2.1 should be

Definition 3.2.1 (Tangent space of a manifold). Let $M \subset \mathbb{R}^n$ be a k-dimensional manifold, so that near $\mathbf{z} \in M$, M is the graph of a C^1 mapping $\mathbf{f}$ expressing $n - k$ variables as functions of the other k variables. If $\mathbf{z} = \mathbf{a} + \mathbf{f}(\mathbf{a})$, then the tangent space to M at $\mathbf{z}$, denoted $T_{\mathbf{z}}M$, is the graph of $[\mathbf{Df}(\mathbf{a})]$.

Page 318 Example 4.2.3, second line of second paragraph: "playing the role of $\mathbf{x}$" (not $\mathbf{x}_1$).

Page 319 Example 3.2.5 refers to Example 3.1.11, but that example concerned a different function.

Page 320 We should have said that $\mathbf{a} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

Page 321 Equation 3.2.19 should end with $= \mathbf{0}$.

Page 325 First line after Equation 3.3.10: the reference should be to Equation 3.3.9 and footnote 7.

Page 326 First line after Equation 3.3.16: "There are 30 such terms" refers to terms *other* than the five terms in Equation 3.3.16. Thus there are 35 in all.

Page 332 The first term in the 4th line should have a minus sign: $-4y\sin(x+y^2)$. Line immediately before Equation 3.3.38: "$(-\frac{1}{3}!)h_1^3$" should be "$(-\frac{1}{3!})h_1^3$."

Page 334 Exercise 3.3.6: in part (b), the hypothesis $f(-\mathbf{x}) = -f(\mathbf{x})$ should have been included. Lines 2 and 3 from bottom: "We will write them only near 0, but by translation they can be written at any point where the function is defined" (not " … they can be written anywhere").

Page 337 The first margin note suggests, incorrectly, that all odd functions and all even functions have Taylor polynomials.

Page 340 Equation 3.4.17 should include $= 0$.

Page 342 Exercise 3.4.4 should read

Find numbers a, b, c such that when f is C^3,

$$h\Big(af(0) + bf(h) + cf(2h)\Big) - \int_0^h f(t)\,dt \in o(h^3).$$

(If you omit the factor h, then a, b, c are not numbers, but multiples of h.)

Exercise 3.4.5 should read

Find numbers a, b, c such that when f is C^3,

$$h\Big(af(0) + bf(h) + cf(2h)\Big) - \int_0^{2h} f(t)\,dt \in o(h^3).$$

Page 343 In the displayed equation in the margin, $Q(t)$ should be $Q(f)$.

Page 345 Equation 3.5.6: the right side should have a "$\pm$" before the square root.

Page 347 The margin note halfway down the page should specify a quadratic form on $\mathbb{R}^n$:

Pages 348–349 In several places – Equations 3.5.23, 3.5.25, and 3.5.28, and in the sentence before Equation 3.5.25 – we write things of the form $\alpha_1(\mathbf{x})^2$ which would be better written with an additional set of parentheses: $(\alpha_1(\mathbf{x}))^2$.

Page 351 Exercise 3.5.1: "and finally the terms in x," not " … in y."

Page 356 Last margin note: This is true for a quadratic form on $\mathbb{R}^n$.

Page 357 First margin note: This is true for a quadratic form on $\mathbb{R}^n$.

Page 360 Exercise 3.6.5, part (a) should read

(a) Find the critical points of the function $f\begin{pmatrix} x \\ y \\ z \end{pmatrix} = xy + yz - xz + xyz$.

Page 366 Third paragraph of Example 3.7.6: The reference should be to Definition 3.1.16, not 3.1.18.

Page 368 Example 3.7.9 contains various errors. For the corrected version, see the complete list of comments and errata.

Page 374 Theorem 3.7.16 should read

Theorem 3.7.16. *A quadratic form Q_A has signature (k, l), if and only if A has k linearly independent eigenvectors with positive eigenvalues and l linearly independent eigenvectors with negative eigenvalues.*

The last margin note should refer to Equation 3.7.55 (not 3.7.54).

Page 376 Exercise 3.7.8: For $a, b \geq 0$. Exercise 3.7.14: We should have said "the unit circle".

Page 378 Equation 3.8.5: $g(X)$, not $g(x)$.

Page 379 Example 3.8.3, next to last line: the curvature $\frac{2}{5\sqrt{5}}$ is about 0.179, not 0.896.

Page 383 The sentence following Definition 3.8.8 should be: Exercise 3.8.3 asks you to show that the absolute value of the mean curvature of the unit sphere is 1 and that the Gaussian curvature of the unit sphere is 1.

Page 384 Caption for Figure 3.8.6: The discussion of the second and third goats should read

"The second goat is thin. He lives on the top of a hill, with positive Gaussian curvature; he can reach less grass. The third goat is fat. His surface has negative Gaussian curvature; with the same length chain, he can get at more grass. This would be true even if the chain were so heavy that it lay on the ground."

Page 387 Equation 3.8.42: The numerator should be $4(a^2 - b^2)$.

Page 392 The fourth line of Equation 3.8.68 should be $= \Big(-\big(\kappa\big(s(t)\big)\big)^2\big(s'(t)\big)^3 + s'''(t)\Big)\vec{\mathbf{t}}(s(t))$.

For consistency, the last line of Equation 3.8.68 should be $\vec{\mathbf{b}}\big(s(t)\big)$, not $\vec{\mathbf{b}}$.

Page 393 Exercise 3.8.3: show that the absolute value of the mean curvature of the unit sphere is 1 and that the Gaussian curvature is 1.

Page 398 Exercise 3.21, part (a): $2d\cos\varphi$ should be $2ad\cos\varphi$.

Chapter 4

Page 412 In Exercise 4.1.5, part (d) and Exercise 4.1.6, part (c), a should be positive: $0 < a < b$.

Page 418 Line above Equation 4.2.10: $\int_0^\pi \sin\theta\, |d\theta| = 2$. (It does not equal π.)

Page 421 Equation 4.2.15: this sums to 2, not 4/3! So in the next sentence, it should be "any sum smaller than \$2 … "

Page 422 There is an unfortunate typo in Equation 4.2.17; a $\mu(\mathbf{x})$ was omitted on the right side. The integral on the right should be $\int_S \big(f(\mathbf{x}) - E(f)\big)^2 \mu(\mathbf{x}) |d^k\mathbf{x}|$.

Page 424 In Equation 4.2.25, the $-t^2$ in the exponent should be $-x^2$: $\mu(x) = \dfrac{1}{\sqrt{2\pi}}\, e^{-x^2/2}$.

Page 426 In the margin note about the error function, the 2π on the left should be $\sqrt{2\pi}$.

Page 428 Exercise 4.2.5: The sample space is all of $\mathbb{R}$. Part (a) should have (x) at the end: $\mu(x) = \dfrac{1}{2a}\chi_{[-a,a]}(x)$.

Page 437 In the last line of the proof of Theorem 4.4.3, the second equation should be

$$\sum_{i,j} \operatorname{vol} B_{i,j} \le \epsilon, \quad \text{not} \quad \operatorname{vol} X_1 \cup X_2 \cup \cdots \le \epsilon$$

Page 438 p.438 Line 10: Example 4.3.3, not 4.4.2.

Page 440 Second paragraph of the proof of Lemma 4.4.6: $|\mathbf{x}_j - \mathbf{y}_j|$, not $|f(\mathbf{x}_j) - f(\mathbf{y}_j)|$: "Since $|\mathbf{x}_j - \mathbf{y}_j| \to 0$ as $j \to \infty$, the subsequence $\mathbf{y}_{j_k}$ also converges to $\mathbf{p}$."

Page 454 Exercise 4.5.17, part (a): "Let $M_r(\mathbf{x})$ be the rth smallest ... ", not largest.

Page 459 In Equation 4.6.14, the sum on the right should start at $i = 1$, not $i = -k$.

Page 465 Exercise 4.6.2: for $k = 1$, we meant the initial conditions to be $x_1 = .7$ and $x_2 = .5$.

Page 467 Definition 4.7.2: We should have specified a *bounded* subset and a *finite* collection.

Page 480 In two places in the first line after Equation 4.8.44, $\operatorname{sgn}(\sigma)$ should be $\operatorname{sgn}(\sigma')$: "and the result follows from $\operatorname{sgn}(\tau^{-1} \circ \sigma') = \operatorname{sgn}(\tau^{-1})\big(\operatorname{sgn}(\sigma')\big) = -\operatorname{sgn}(\sigma')$, since ... "

Page 485 Last line of first paragraph: "volume of the parallelepiped," not area.

Page 488 In the equation following Equation 4.9.12, the left side should be

$$U_{T(\mathcal{D}_N)}(\chi_{T(A)}) - L_{T(\mathcal{D}_N)}(\chi_{T(A)}).$$

Page 494 Discussion after Proposition 4.10.3: Corollary 4.3.10, not Theorem 4.3.9.

Page 496 Last margin note: The sentence "At $\varphi = -\pi/2$ and $\varphi = \pi/2$, $r = 0$" should be deleted.

Page 502 Line 5: "in this case we can solve $xy = u$", not "in this case we can solve $y = u/v$".

Page 502 Bottom margin note: Exercise 4.10.4, not 4.5.19.

Page 503 Exercise 4.10.3: This exercise should read "Show that in complex notation, with $z = x + iy$, the equation of the lemniscate of Figure 4.10.3 can be written $|z^2 - \frac{1}{2}| = \frac{1}{2}$. Hint: See Example 4.10.19."

Page 509 The proof of Theorem 4.11.8 is not correct; the main idea is right but there is a fiddly problem with the truncations. For a rewritten proof, see the complete list of errata or the errata list for Chapter 4.

Page 510 Equation 4.11.19: if $[f_k]_R = f_k$ and $[g_k]_r = g_k$, this does not imply $[f_k - g_k]_R = f_k - g_k$. The simplest way to fix this seems to be to change Equation 4.11.19, stating explicitly that we are choosing R big enough so that:

$$\sum_{k=1}^m f_k = \sum_{k=1}^m [f_k]_R, \qquad \sum_{k=1}^m g_k = \sum_{k=1}^m [g_k]_R, \qquad \sum_{k=1}^m f_k - g_k = \sum_{k=1}^m [f_k - g_k]_R. \qquad 4.11.19$$

The left side of Equation 4.11.22 should be an absolute value.

Page 514 The statement in the margin that "the union of sets of measure 0 has measure 0" is incorrect. It should be "the union of finitely many (or countably many) sets of measure 0 has measure 0."

Page 514 The proof of Proposition 4.11.14 is not correct. For a correct proof, see the complete list of comments and errata.

Page 518 Last line in the margin, the third integral should be $i \displaystyle\int f_2(x)\,dx$ (i.e., f_2 not f_1).

Page 520 Equation 4.11.73: An integral sign $\int_0^\infty$ is missing on the right of the second line.

Page 526 A sum sign is missing in Exercise 4.27: $f(x) = \displaystyle\sum_{k=1}^{\infty} \frac{1}{2^k} \frac{1}{\sqrt{|x-a_k|}}$.

Chapter 5

Page 534 Equation 5.2.4: a_1 should be a_i in two places, and the "for $a_1, a_2, a_3, \dots$ should be omitted:

$$U = \bigcup_{i=1}^{\infty} \left(a_i - \frac{1}{2^{N+i}},\ a_i + \frac{1}{2^{N+i}} \right). \qquad 5.2.4$$

The next sentence should say "This is an open subset of $\mathbb{R}$ … ," not "This is an open subset of $[0,1]$ … ."

In Equation 5.2.5, the sum should start at $n = 1$ not $= 1$. On the right sides of Equations 5.2.5 and 5.2.6, the denominator should be 2^{N-1}, not 2^{N-2}.

Page 537 Middle margin note: z-axis, not x-axis.

Page 545 line should end with $d\theta$. Equation 5.3.27: This equation should not have a $d\theta$ at the end.

Page 549 In three places, $D_2 f$ should be $D_3 f$ (for details, see the complete list of comments and errata).

Page 552 First margin note: the earth's circumference, not diameter! Exercise 5.3.12: The total curvature of a curve C is $\int_C \kappa\, |d^1\mathbf{x}|$.

Page 556 Exercise 5.6: Some subscripts were omitted, and one superscript is wrong. It should be:

(a) Show that $w'_{n+1}(r) = v_n(r)$.

(b) Show that $v_n(r) = r^n v_n(1)$.

(c) Derive Equation 5.3.49, using $w_{n+1}(1) = \int_0^1 w'_{n+1}(r)\,dr$.

Chapter 6

Page 562 The right side of Equation 6.1.14 should be $\displaystyle\sum_{i=1}^{k-1} a_i \varphi(\vec{\mathbf{v}}_1, \dots, \vec{\mathbf{v}}_{k-1}, \vec{\mathbf{v}}_i)$.

Page 564 A dx is missing in (6.1.23); it should be $dx_{i_1} \wedge \cdots \wedge dx_{i_k}(\vec{\mathbf{e}}_{j_1}, \dots \vec{\mathbf{e}}_{j_k})$.. Equation 6.1.24 should be

$$dx_{j_1} \wedge \cdots \wedge dx_{j_k}(\vec{\mathbf{e}}_{j_1}, \dots, \vec{\mathbf{e}}_{j_k}) = 1. \qquad 6.1.24$$

Page 571 Exercise 6.1.2 (a): $dx_3 \wedge dx_2 \wedge x_4$ should be $dx_3 \wedge dx_2 \wedge dx_4$.

Page 581 As written, Proposition 6.3.5 assumes that an appropriate normal vector field can be chosen; this is not always the case. For the rewritten statement and proof, see the complete list of comments and errata.

Page 582 Proposition 6.3.8: We should have said "Suppose there exists a normal vector field $\vec{\mathbf{n}}$", not "Choose a normal vector field $\vec{\mathbf{n}}$". If no normal vector field $\vec{\mathbf{n}}$ exists, then the manifold is not orientable.

Page 584 In Equation 6.3.9, the second equality is incorrect; the second determinant is opposite the first. See the complete list of comments and errata for the rewritten example.

Page 590 Exercise 6.3.17, part (b): The curve C should be smooth.

Page 591 Third margin note: Definition 6.4.2, not 6.4.1.

Page 595 Second line in margin: pullback of ω, not pullback of φ.
Margin note half-way down the page: Equation 6.4.20, not 6.4.19.

Page 603 Caption for Figure 6.5.2: In two places, $x\,dx + y\,dx$ should be $x\,dx + y\,dy$.

Page 603 The sentence "the requirement of antisymmetry then says that $f(-P_{\mathbf{x}}) = -f(\mathbf{x})$" should be deleted.

Page 605 Figure 6.5.7: the vector field should turn clockwise.

Page 606 Line 4: clockwise, not counter-clockwise.

Page 607 Example 6.5.6: the 3rd entry of the tangent vector field is 1, not 0. In Equation 6.5.13, the third entry of $\vec{\gamma}'(t)$ is 1, not 0; the dot product is 2.

Page 609 Last margin note: the signs are reversed in the matrix; it should be $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

Page 619 Equation 6.6.5: the first equation should be $g(\mathbf{y}) = (\mathbf{y} - \mathbf{0}) \cdot \vec{\mathbf{w}}_i = 0$.

Page 622 Last line of Definition 6.6.10: $\partial_1 P$ should be $\partial_M P$.

Page 626 Exercise 6.6.5: $\vec{\nabla}$ denotes the transpose of the derivative: $\vec{\nabla} f(\mathbf{x}) = [\mathbf{D}f(\mathbf{x})]^\top$.

Page 627 Exercise 6.6.8 should say that M is oriented by $dx_1 \wedge dx_2 \wedge dx_3$. In Exercise 6.6.8, "at a point of ∂M" should be "at a point of ∂X".

Page 631 In the first line of Equation 6.7.14, φ should be ψ.

Page 632 In Theorem 6.7.7, we should have said, "For any k-form φ of class C^2 . . . ."

Page 633 Second margin note: Theorem 6.7.8, not A6.7.8.

Page 636 The last term on the right-hand side of Equation 6.8.5 should be $D_3 f v_3$, not $D_3 v_3$.

Page 640 In the margin note, curl $\vec{F}$ should be curl curl $\vec{F}$.

Page 651 Theorem 6.10.2: "Definition 6.6.13" should be "Definition 6.6.10".

Page 657 Exercise 6.10.8 is wrong as written; indeed, it contradicts Exercise 6.10.7. The vector fields should be $\begin{bmatrix} xy^2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ -x^2 y \end{bmatrix}$.

Page 658 Exercise 6.10.15, part (b): "of equation $z_1^p + z_2^q = 0$, not "of equation $z_1^p + z_2^q$"."

Page 659 We should have chosen our bicycle trip at the top of the hill; then it would be clear that if a cyclist starts and ends at the same point, he or she does no work against gravity.

Page 661 Margin note: Equation 6.5.12, not 5.6.1.

Page 664 Exercise 6.1.3, part (b): "Sketch the potential" should be "sketch the electric field."

Page 665 Exercise 6.11: "for the following 1-forms on $\mathbb{R}^2$ should be "for the following 1-forms."

Page 666 Exercise 6.12: the matrix should be $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. This affects parts (a) and (b).

Page 667 In Exercise 6.18, part (b), the displayed equation should be $\mathrm{vol}_n\big(B_1^n(\mathbf{0})\big) = \frac{1}{n}\,\mathrm{vol}_{n-1}(S^{n-1})$.

Appendix A

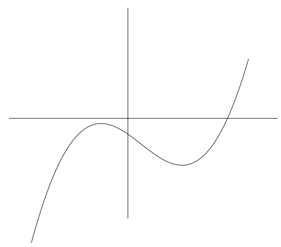
Page 670 In the first sentence after Definition A1.2, $\mathrm{Assoc}(x, y) = (x + y) + z$ should be $\mathrm{Assoc}(x, y, z) = (x + y) + z$.

The words "k-close" were omitted from Definition A1.3, which should read "Two points $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ are k-close if for each $i = 1, \dots, n$, then $\big|[x_i]_k - [y_i]_k\big| \le 10^{-k}$."

Page 671 Exercise A1.2 left out "$\mathrm{Assoc}(x, y, z) =$." The first sentence of the exercise should read

"Show that the functions $A(x, y) = x + y$, $M(x, y) = xy$, $S(x, y) = x - y$, and $\mathrm{Assoc}(x, y, z) = (x + y) + z$ are $\mathbb{D}$-continuous, and that $1/x$ is not."

The bottom graph in Figure A2.1 is wrong; it should be:



Page 682 Restatement of Theorem 2.7.13: in the next-to-last line, it should be "has a unique solution in the closed ball $\overline{U_0}$".

Page 691 We corrected Equation 2.9.13 on page 270. Of course it should also be corrected here.

Page 692 The proof of Theorem 2.9.7 does not include a proof of the last statement, concerning Equation 2.9.13. For the missing proof, see the complete errata list or the list for the Appendix.

Page 695 Equation A8.6 is wrong. It should be $\mathbf{F}\begin{pmatrix} \mathbf{g}(\mathbf{y}) \\ \mathbf{y} \end{pmatrix} = \mathbf{0}$.

Page 707 Equation A12.3 should end with ds, not dt.

Page 723 In the next-to-last line of the paragraph beginning "Fortunately", the word "volume" should be "measure".

Page 724 Corollary A16.3: "measure", not "volume". Seeing why the proof is correct requires the following corollary to Theorem 4.4.5: If f and g are integrable functions on $\mathbb{R}^n$, $g \ge f$, and $\int f(\mathbf{x})|d^n\mathbf{x}| = \int g(\mathbf{x})|d^n\mathbf{x}|$, then $\{\, \mathbf{x} \mid f(\mathbf{x}) \ne g)\mathbf{x})\,\}$ has measure 0. We propose making this into an exercise, with the hint: Show that if $g(\mathbf{x}_0) > f(\mathbf{x}_0)$ and $g - f$ is continuous at $\mathbf{x}_0$, then $\int g(\mathbf{x})|d^n\mathbf{x}| > \int f(\mathbf{x})|d^n\mathbf{x}|$. Then apply Theorem 4.4.5.

Page 727 It is possible that $\mathbf{x}$ may be in more than one tile. By Corollary 4.3.10, such points don't affect integals; however, the definition of $\overline{g}$ should take such points into account. For details, see the complete list of errata and comments.

Pages 742, 743, 745 Each page has "integrable functions" that should be "R-integrable functions".

Page 744 The last sum in Equation A21.8 should have i, not k: $\sum_{i=1}^{\infty} \int h_i |d^n\mathbf{x}|$.

Page 748 Equation A21.26: In the first line, the sums should be over $C \subset Y_0$, not $C \in Y_0$. But then we also have to specify that the C are in $\mathcal{D}_{N_0}(\mathbb{R}^n)$. This gives

$$\begin{aligned}\mathrm{vol}_n(Y_0)\frac{A}{\epsilon} = \sum_{\substack{C\subset Y_0 \\ C\in\mathcal{D}_{N_0}(\mathbb{R}^n)}} \frac{A}{\epsilon}\,\mathrm{vol}_n(C) \;\le\; \sum_{\substack{C\subset Y_0 \\ C\in\mathcal{D}_{N_0}(\mathbb{R}^n)}} M_C(h_0)\,\mathrm{vol}_n(C) \\ \le \sum_{C\in\mathcal{D}_{N_0}} M_C(h_0)\,\mathrm{vol}_n(C) = U_{N_0}(h_0) \le 2A,\end{aligned} \qquad \text{A21.26}$$

In Equation A21.29, h_m comes with a + sign and h_{m+1} with a − sign; it should be reversed. In the second line, the = should be <. So the equation should read

$$\begin{aligned}\int_{\mathbb{R}^n} g_{m+1}(\mathbf{x})|d^n\mathbf{x}| = \left(\int_{\mathbb{R}^n} h_{m+1}(\mathbf{x})\,|d^n\mathbf{x}| - A\right) - \left(\int_{\mathbb{R}^n} h_m(\mathbf{x})\,|d^n\mathbf{x}| - A\right) \\ \le \frac{A}{4^{m+3}} + \frac{A}{4^{m+2}} < \frac{A}{2\cdot 4^{m+1}}.\end{aligned} \qquad \text{A21.29}$$

Page 749 Equation A21.35 : on the far right, the A in the numerator should be ϵ.

Page 750 The equation in the footnote contains typos involving the absolute value signs and parentheses. For the correct version, see the complete list of comments and errata.

Page 753 Two lines after Equation A21.49 replace "volume 0" by "measure 0" in two places. Sentence right after Equation A21.50: third and fourth "equalities", not "inequalities."

Page 754 Third displayed equation: the bracket on the left should say "finite because f, hence g, is L-integrable." In the paragraph beginning "For the converse", $\mathbb{R}^n$ should be $\mathbb{R}^{n+m}$. Even with that correction, we were not quite rigorous in arguing that $f\chi_C$ is L-integrable. Another version is given in the complete list of errata and notes.

Page 755 We say we prove the "if" part in the text, leaving "if only" as an exercise. It's the reverse.

Page 755 Last line: "Exercise A21.2" should be "Exercise A21.5."

Page 759 Last exercise of the section: prove the "if" part, not "only if".

Page 766 Definition A24.1 should read

Definition A24.1 (Pullback by a linear transformation). Let V, W be vector spaces, and $T : V \to W$ be a linear transformation. Then T^* is a linear transformation $A^k(W) \to A^k(V)$, defined as follows: if φ is a k-form on W, then $T^*\varphi$ is the k-form on V given by

$$T^*\varphi(\vec{\mathbf{v}}_1, \dots, \vec{\mathbf{v}}_k) = \varphi\big(T(\vec{\mathbf{v}}_1), \dots, T(\vec{\mathbf{v}}_k)\big). \qquad \text{A24.1}$$

Page 769 In the last line of Equation A24.14, the $\mathbf{g}^*\mathbf{f}^*$ should be $\mathbf{f}^*\mathbf{g}^*$.

Page 770 In the first line of Equation A24.18 (last term), $[\mathbf{D}g(\mathbf{x})]$ should be $[\mathbf{Df}(\mathbf{x})]$.

Equation A24.19: "Theorem A6.7.8" should be "Theorem 6.7.8."

Page 782 In Exercise A25.2, "(proof of Lemma A25.12)" should be "(see Equation A25.12)".

Inside back cover The "useful formulas: trigonometry" would be more useful if they were all correct! Sorry! The fourth and fifth formulas should be

$$\cos\alpha = \sin(\pi/2 - \alpha) \quad \text{and} \quad \sin\alpha = \cos(\pi/2 - \alpha).$$

Index

Page 792 dominated convergence (Lebesgue), 515 (not 516)

The listing for *diffeomorphism* on page 514 should be deleted.