Errata and Clarifications: Preface and Chapter 0

Updated September 18, 2004

Preface

Page xv 16th line from bottom: "some of the material," not "some of material."

Chapter 0

Page 4 Second and third lines from the bottom: To be punctilious, we should have said "for all $y \in \mathbb{R}$:

"For all $x \in \mathbb{R}$ and for all $\epsilon > 0$, there exists $\delta > 0$ such that for all $y \in \mathbb{R}$, if $|y - x| < \delta$, then $|y^2 - x^2| < \epsilon$."

Page 4 Margin note about Bowditch: "American Bowditch," not "America Bowditch."

Page 5 There are some inconsistencies of notation. In future editions we will write Equation 0.2.2 as

$$\text{The opposite of} \quad (\forall\, x)P(x) \quad \text{is} \quad (\exists x) \text{ not } P(x).$$

(But if we had something more complicated than $P(x)$ we would put it in parentheses.)

Most mathematicians avoid the symbolic notation, instead writing out quantifiers in full. But when there is a complicated string of quantifiers, they often use the symbolic notation to avoid ambiguity.

Page 7 It follows from the definition of $\subset$ that the empty set ϕ is a subset of every set.

We should have included an eighth "word":

$=$ "equality"; $A = B$ if A and B have the same elements

and specified that the symbol $\notin$ ("not in") means "not an element of"; similarly, $\not\subset$ means "not a subset of" and $\neq$ means "not equal".

We should also have noted that the order in which elements of a set are listed (even assuming they are listed) does not matter, and that duplicating does not affect the set; for example, $\{1, 2, 3\} = \{1, 2, 3, 3\}$.

Page 12 Example 0.4.4: The first sentence after the displayed equation should be

"This can be evaluated only if $x^2 - 3x + 2 \geq 0$, which happens if $x \leq 1$ or $x \geq 2$."

($x \leq 1$, not $x \leq 2$; $x \geq 2$, not $x \geq 3$.)

The second sentence should be

"So the natural domain is $(-\infty, 1] \cup [2, \infty)$."

Page 15 Example 0.4.10: The range of f should be the real numbers, not the real positive numbers. (The *image* of f is of course the real positive numbers.) Speaking of $f^{-1}(A)$ does not require, or even suggest, that A is a subset of the image of f. As a rule, finding the image of f is difficult, and it would drastically restrict the language of $f^{-1}(A)$ to make such a requirement.

Page 18 First line: "in this section" should be "in this section and in Appendix A.1."

Definition 0.5.1: The least upper bound is also known as the supremum. Definition 0.5.2: The great lower bound is also known as the infimum.

Page 18 The definition of the truncation $[x]_k$ and the discussion in the following paragraph of one number being larger than another fails to take into account the non-fractional part. Here is the rewritten version:

We denote by $[x]_k$ the number formed by keeping all digits to the left of and including the 10^{-k} column and setting all others to 0. Thus if $x = 5\,129.359\ldots$, then $[x]_{-2} = 5\,100.00\ldots$, $[x]_{-1} = 5\,120.00\ldots$, $[x]_0 = 5\,129.00\ldots$, $[x]_1 = 5\,129.30\ldots$ and so on. To avoid ambiguity, If x is a real number with two decimal expressions, $[x]_k$ will be the finite decimal built from the infinite decimal ending in 0's; for the number in Equation 0.5.1, $[x]_3 = 0.350$; it is not 0.349.

Given any two different finite numbers x and y, one is always bigger than the other. This is defined as follows. If x is positive and y is nonpositive, then $x > y$. If both are positive, then in their decimal expansions there is a leftmost digit in which they differ; whichever has the larger digit in that position is larger. If both x and y are negative, then $x > y$ if $-y > -x$.

Page 19 The proof of Theorem 0.5.3 also fails to take into account the nonfractional part. The second paragraph of the proof should be replaced by the following:

Let the kth digit of a number be the digit in the 10^{-k} column. Thus if $k = -2$, the kth digit of 237.05 is 2; if $k = 0$, the kth digit is 7. Since $x \neq a$, there is then a smallest j such that $[x]_j < [a]_j$. There are 10 numbers that have the same kth digit as x for $k < j$ and that have 0 as the kth digit for $k > j$; consider those that are in $[\,[x]_j, a]$. This set is not empty, since $[a]_j$ is one of them. Let b_j be the largest such that $X \cap [b_j, a] \neq \phi$; such a b_j exists, since $x \in X \cap [[x]_j, a]$.

Now consider the set of numbers in $[b_j, a]$ that have the same kth digit as b_j for $k < j+1$, and 0 for $k > j+1$. Again this is a nonempty set with at most 10 elements, and b_j is one (the smallest) of them. Call b_{j+1} the largest such that $X \cap [b_{j+1}, a] \neq \phi$. Again such a b_{j+1} exists, since if necessary we can choose b_j. Keep going this way, defining numbers b_{j+2}, b_{j+3}, and so on, and let b be the number whose nth decimal digit (for all n) is the same as the nth decimal digit of b_k. We claim that $b = \sup X$.

Page 20 Last line of the proof of Theorem 0.5.7: $A - a_n \leq A - a_N$, not $A - a_n < A - a_N$.

Page 21 In the first line of the proof of Theorem 0.5.8, the summmand should be in parentheses: $\sum_{n=1}^{\infty}(a_n + |a_n|)$

Exercise 0.5.1: Exercise 1.6.11 repeats this exercise, with hints.

Page 23 On lines 7–8 we say that sets that can be put in one-to-one correspondence with the integers are called countable.

Two lines before Equation 0.6.4:

"of degree <= 2 with $|ai|$ <= 2", not " ... with ai <= 2".

At the very bottom of the page we say that "a set A is countable if $A \asymp \mathbb{N}$, and ... ". The natural numbers can be put in one-to-one correspondence with the integers, so there is no actual error, but in subsequent editions we will change "integers" to "natural numbers".

Page 24 Second full paragraph: $\mathcal{P}(E)$ is called the *power set* of E.

In the same paragraph, we use the symbol $\mapsto$, which is not explained until page 71. This can be avoided by some rewriting:

Clearly for any set E there exists a one-to-one map $f : E \to \mathcal{P}(E)$; for instance, the map $f(a) = \{a\}$.

Page 25 Exercise 0.6.6: Our solution does not actually use the hint. You can use Bernstein's theorem, but it seems a little harder than proving the result directly.

Page 31 Exercise 0.7.4: "Of the following complex numbers," not "of of the . . . ."

Page 32 Exercise 0.7.10: the word "number" should be plural: "Describe the set of all complex numbers".