Vector Calculus, Linear Algebra, and Differential Forms: A Unified Approach

Errata and Clarifications for the Second Edition

Chapter 1

updated October 28, 2004

Page 40 In the margin note beginning "The trivial subspace," the **0** should be $\vec{\mathbf{0}}$.

Page 45 Sentence immediately before Definition 1.2.4: "I If" should be "If."

Page 48 Figure 1.2.5: On the right, the final matrix should be written $A(BC)$, not $(AB)C$.

Page 52 The paragraph immediately before Example 1.2.21 should not be there; it was printed twice.

Page 57 Exercise 1.2.17 should read "Recall ... what the adjacency matrix of a graph is" (not "what the adjacency graph of a matrix is").

Page 61 In the first paragraph of the remark, the mention of "feedback" is incorrect. Feedback is compatible with linearity. The end of the paragraph, beginning with "Nor do linear transformations allow for feedback," has been rewritten as follows:

Nor does a linear model of the "price transformation" allow for the possibility that if you buy more you will get a discount for quantity, or that if you buy even more you might create scarcity and drive prices up. The failure to take such effects into consideration is a fundamental weakness of all models that linearize mappings and interactions.

Page 65 In Theorem 1.3.11, we assumed that the composition $T \circ S$ is a linear transformation. We should have stated this as part of the theorem and then proved it.

Thus the theorem should read

Theorem 1.3.11 (Composition corresponds to matrix multiplication). *Suppose $S : \mathbb{R}^n \to \mathbb{R}^m$ and $T : \mathbb{R}^m \to \mathbb{R}^l$ are linear transformations given by the matrices $[S]$ and $[T]$ respectively. Then the composition $T \circ S$ is linear and*

$$[T \circ S] = [T][S]. \qquad 1.3.13$$

The proof should begin with a proof of linearity:

The following computation shows that $T \circ S$ is linear:

$$\begin{aligned}(T \circ S)(a\vec{\mathbf{v}} + b\vec{\mathbf{w}}) &= T\big(S(a\vec{\mathbf{v}} + b\vec{\mathbf{w}})\big) = T\big(aS(\vec{\mathbf{v}}) + bS(\vec{\mathbf{w}})\big) \\ &= aT\big(S(\vec{\mathbf{v}})\big) + bT\big(S(\vec{\mathbf{w}})\big) = a(T \circ S)(\vec{\mathbf{v}}) + b(T \circ S)(\vec{\mathbf{w}}).\end{aligned}$$

Page 71 Exercise 1.3.22 belongs in Section 1.4, as it uses the dot product and orthogonality.

Page 76 The proof of Theorem 1.4.5 should start with the sentence "If either $\vec{\mathbf{v}}$ or $\vec{\mathbf{w}}$ is $\mathbf{0}$, the statement is obvious, so suppose both are nonzero."

Page 79 Figure 1.4.9: $\mathbf{h}$ should be h, the height of the parallelogram.

Page 85 Figure 1.4.12: The arc is misplaced. It should go from $\mathbf{h}$ to $\mathbf{a}$, as shown below.

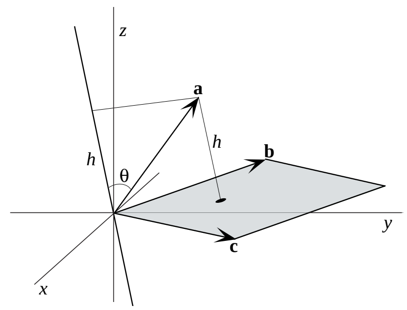

FIGURE 1.4.12.

Page 93 In Equation 1.5.13, U should be $\overline{U}$.

Page 94 Third line: "an incontestably correct definition," not "a incontestably correct definition."

Page 98 Definition 1.5.20 of a limit of a function should have concerned a function $\mathbf{f} : X \to \mathbb{R}^m$. In the definition, the discussion, and the proof of Proposition 1.5.21, every f should be $\mathbf{f}$, a should be $\mathbf{a}$, and b should be $\mathbf{b}$.

Page 100 Theorem 1.5.23: We should have specified that U is a subset of $\mathbb{R}^n$. In Equation 1.5.35, we should have written $(h\mathbf{f})(\mathbf{x})$, not $h\mathbf{f}(\mathbf{x})$.

Page 101 Proof of Theorem 1.5.23: since $\mathbf{x}_0$ is not guaranteed to be in U, we should replace $\mathbf{f}(\mathbf{x}_0)$ by $\mathbf{a} := \lim_{\mathbf{x}\to\mathbf{x}_0} \mathbf{f}(\mathbf{x})$ and $\mathbf{g}(\mathbf{x}_0)$ by $\mathbf{b} := \lim_{\mathbf{x}\to\mathbf{x}_0} \mathbf{g}(\mathbf{x})$.

Page 101 The notation in the proof of Theorem 1.5.24 is a little mixed up. The theorem and proof should read

Theorem 1.5.24 (Limit of a composition). *If $U \subset \mathbb{R}^n$, $V \subset \mathbb{R}^m$ are subsets, and $\mathbf{f} : U \to V$ and $\mathbf{g} : V \to \mathbb{R}^k$ are mappings, so that $\mathbf{g} \circ \mathbf{f}$ is defined in U, and if $\mathbf{y}_0 := \lim_{\mathbf{x}\to\mathbf{x}_0} \mathbf{f}(\mathbf{x})$ and $\mathbf{z}_0 := \lim_{\mathbf{y}\to\mathbf{y}_0} \mathbf{g}(\mathbf{y})$ both exist, then $\lim_{\mathbf{x}\to\mathbf{x}_0} \mathbf{g} \circ \mathbf{f}(\mathbf{x})$, exists, and*

$$\lim_{\mathbf{x}\to\mathbf{x}_0} (\mathbf{g} \circ \mathbf{f})(\mathbf{x}) = \mathbf{z}_0. \qquad 1.5.45$$

Proof. For all $\epsilon > 0$ there exists $\delta_1 > 0$ such that if $|\mathbf{y} - \mathbf{y}_0| < \delta_1$, then $|\mathbf{g}(\mathbf{y}) - \mathbf{z}_0| < \epsilon$. Next, there exists $\delta > 0$ such that if $|\mathbf{x} - \mathbf{x}_0| < \delta$, then $|\mathbf{f}(\mathbf{x}) - \mathbf{y}_0| < \delta_1$. Hence

$$|\mathbf{g}(\mathbf{f}(\mathbf{x})) - \mathbf{z}_0| < \epsilon \quad \text{when} \quad |\mathbf{x} - \mathbf{x}_0| < \delta. \quad \square \qquad 1.5.46$$

Page 102 Line 3: "the limit does not exist", not "the limit may not exist".

Page 103 Theorem 1.5.28 (e): "even if $\mathbf{f}$ is not continuous at $\mathbf{x}_0$", not "even if $\mathbf{f}$ is not defined at $\mathbf{x}_0$".

Margin note, end of first paragraph: "for different values of $\mathbf{x}_0$", not "for different values of $\mathbf{x}$".

Page 105 The proof of Proposition 1.5.34 should read

Set $\vec{\mathbf{a}}_i = \begin{bmatrix} a_{1,i} \\ \vdots \\ a_{n,i} \end{bmatrix}$. Then $|a_{k,i}| \leq |\vec{\mathbf{a}}_i|$, so $\sum_{i=1}^{\infty} |a_{k,i}|$ converges, so by Theorem 0.5.8, $\sum_{i=1}^{\infty} a_{k,i}$ converges. Proposition 1.5.34 then follows from Proposition 1.5.13.

Last margin note: "Newton's method," not "Newton's method's."

Page 106 Third line of proof of Proposition 1.5.35: $S_k(I - A) = I - A^{k+1}$, not $S_l(I - A) = I - A^{k+1}$.

Note: One vigilant reader objected to Equation 1.5.60; how do we know that $\lim_{k\to\infty} S_k(I - A)$ exists? To be perfectly rigorous, we should have written the equation in the opposite direction, starting with $I = I - \lim_{k\to\infty} A^{k+1}$; then each step is justified.

Page 108 Exercise 1.5.12: We must assume $u(\epsilon) > 0$.

Page 109 Exercise 1.5.19 belongs with exercises for Section 1.6

Exercise 1.5.20, part (a): n was used with two different meanings. Below we changed n to k in two places:

(a) Let Mat (n, m) denote the space of $n \times m$ matrices, which we will identify with $\mathbb{R}^{nm}$. For what numbers $a \in \mathbb{R}$ does the sequence of matrices $A^k \in$ Mat $(2, 2)$ converge as $k \to \infty$, when $A = \begin{bmatrix} a & a \\ a & a \end{bmatrix}$? What is the limit?

Pages 109–110 Exercise 1.5.21, part (d) and Exercise 1.5.24 are very similar.

Page 110 First line of Section 1.6: "the existence of a convergent subsequence", not "the existence of a convergence subsequence".

Page 111 The subheading at the top of the page should be "The existence of a *convergent* subsequence in a compact set", not "the existence of a *convergence* subsequence . . . ".

Page 113 Six lines from the bottom, "the digits $0, 1, 1, 2, 4$" should be "the digits $0, 1, 2, 3, 4$"

Page 123 4th line from bottom: "of degree 1 or 2", not "of degree 1."

Page 125 Hint for Exercise 1.6.7: "minimum" not "maximum."

Page 126 It is incorrect to ascribe the motions of a pendulum to feedback.

Page 133 Equation 1.7.19: the 0 in $\lim \vec{\mathbf{h}} \to 0$ should be bold, since it is a vector.

Page 137 Equation 1.7.37 should have some parentheses:

$$\lim_{h \to 0} \frac{1}{h}\Big(\overbrace{(1+h)(1+2h)\sin(\frac{\pi}{2}+h)}^{\mathbf{f}(\mathbf{a}+h\vec{\mathbf{v}})} - \overbrace{(1 \cdot 1 \cdot \sin\frac{\pi}{2})}^{\mathbf{f}(\mathbf{a})}\Big).$$

Page 138 Remark: In several places we wrote $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ when we meant $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$. The $\vec{\mathbf{v}}$ in the expression $\Big[\mathbf{D}f \begin{bmatrix} 0 \\ 0 \end{bmatrix}\Big]\vec{\mathbf{v}}$ does not belong there. The last half of the remark should read:

. . . to a step of length $\sqrt{5}$ in the direction $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$. To take a step of length 1 in that direction, starting at the origin, we would multiply $\Big[\mathbf{D}f \begin{bmatrix} 0 \\ 0 \end{bmatrix}\Big]$ by $\begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$, which has length 1, to get a rate of ascent (at time 0) of $19/\sqrt{5} \approx 8.5$. In which direction is the function increasing faster, $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ or $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$?

In the footnote, $36/5 \approx 7.2$ should be $36/5 = 7.2$.

Pages 140 and 141 The last margin note on page 140 is almost identical to the first margin note on page 141.

Pages 143 and 145 The running heads at the top of the page should say "1.7 Differential Calculus," not "1.6 Four Big Theorems" and "1.8 Rules for Computing Derivatives."

Page 144 Line 4: "by direct computation", not "by direction computation".

Page 147 Last line: following the equation, we should perhaps add "i.e., for every $\vec{\mathbf{h}} \in \mathbb{R}^n$ we have $[\mathbf{Df}(\mathbf{a})]\vec{\mathbf{h}} = \mathbf{f}(\vec{\mathbf{h}})$."

Page 149 Long displayed equation after Equation 1.8.15, line 2: in the denominator at far right, $(f(\mathbf{a}))^2$ should be $(f(\mathbf{a}))$. In line 3, the notes in underbrackets could be more precise:

$$= \frac{1}{|\vec{\mathbf{h}}|}\left(\frac{f(\mathbf{a}) - f(\mathbf{a}+\vec{\mathbf{h}}) + [\mathbf{D}f(\mathbf{a})]\vec{\mathbf{h}}}{\big(f(\mathbf{a})\big)^2}\right) - \frac{1}{|\vec{\mathbf{h}}|}\left(\frac{f(\mathbf{a}) - f(\mathbf{a}+\vec{\mathbf{h}})}{\big(f(\mathbf{a})\big)^2} - \frac{f(\mathbf{a}) - f(\mathbf{a}+\vec{\mathbf{h}})}{f(\mathbf{a}+\vec{\mathbf{h}})\big(f(\mathbf{a})\big)}\right)$$

$$= \underbrace{\frac{1}{|\vec{\mathbf{h}}|}\left(\frac{f(\mathbf{a}) - f(\mathbf{a}+\vec{\mathbf{h}}) + [\mathbf{D}f(\mathbf{a})]\vec{\mathbf{h}}}{\big(f(\mathbf{a})\big)^2}\right)}_{\text{lim as } h\to 0 \text{ is 0 by def. of deriv.}} - \underbrace{\frac{1}{|\vec{\mathbf{h}}|}\frac{f(\mathbf{a}) - f(\mathbf{a}+\vec{\mathbf{h}})}{f(\mathbf{a})}}_{\text{bounded}}\underbrace{\left(\frac{1}{f(\mathbf{a})} - \frac{1}{f(\mathbf{a}+\vec{\mathbf{h}})}\right)}_{\text{lim as } h\to 0 \text{ is 0}}$$

Page 149 Next to last margin note: "in all mathematics", not "in all of all mathematics".

Page 151 Equation 1.8.22: in the second matrix on right-hand side, the second entry in the third row should be 2, not 1:

$$\left[\mathbf{D}(\mathbf{f}\circ\mathbf{f})\begin{pmatrix}1\\1\\1\end{pmatrix}\right] = \begin{bmatrix}0 & 0 & 4\\ -2 & 1 & 0\\ 0 & 2 & 0\end{bmatrix}\begin{bmatrix}0 & 0 & 2\\ 1 & 1 & 0\\ 0 & 2 & 0\end{bmatrix} = \begin{bmatrix}0 & 8 & 0\\ 1 & 1 & -4\\ 2 & 2 & 0\end{bmatrix}. \qquad 1.8.22$$

Page 154 Exercise 1.8.10: The function $\mathbf{f} : \mathbb{R}^2 \to \mathbb{R}^2$ should be the function $f : \mathbb{R}^2 \to \mathbb{R}$:

Page 157 Parentheses should be added to Equation 1.9.15:

$$\lim_{x\to 0}\left(\frac{1}{2} + 2x\sin\frac{1}{x}\right) = \frac{1}{2}.$$

Page 161 The = in the first line of Equation 1.9.25 should be $\leq$.

Exercise 1.9.1 is identical to Example 1.9.4.

Page 165 Exercise 1.22 is identical to Exercise 1.5.19, which in any case should be in Section 1.6.