

Contents

Foreword by Richard Varga	xi
Preface	xiii
0 Introduction to iterative methods and preconditioning	1
1 Numerical linear algebra: background	5
1.1 Review of matrix theory	5
1.1.1 Basic matrix operations	7
1.1.2 Matrix partitionings	9
1.1.3 Basic concepts of matrix analysis	10
1.1.4 Vector norms and matrix norms	12
1.1.5 Computational work	15
1.2 Eigenvalues and eigenvectors	15
1.2.1 Relating norms and eigenvalues	18
1.2.2 Convergence of vector and matrix sequences	18
1.2.3 Perron-Frobenius theory of nonnegative matrices	24
1.2.4 Diagonally dominant matrices	28
1.2.5 Power method	29
1.3 General linear systems	32
1.3.1 Direct methods	36
1.3.2 Iterative refinement	40
2 The theory of matrix splitting	41
2.1 General properties of matrix splittings	41
2.2 Regular splittings	46
2.3 Nonnegative and weak nonnegative splittings	52
2.4 Weak and weaker splittings	61
2.5 Summary	68
3 Discretization of partial differential equations	70
3.1 Finite-difference approximations	71
3.2 One-dimensional problems	73
3.2.1 Forward elimination – backward substitution	75
3.2.2 Backward elimination – forward substitution	76
3.3 Band matrices	78
3.3.1 Pentadiagonal matrices	78

3.3.2	General band systems	80
3.4	Two-dimensional problems	83
3.4.1	Rectangular geometry	83
3.4.2	Triangular geometry	94
3.4.3	Hexagonal geometry	96
3.4.4	Reduced systems	101
3.4.5	Line orderings	114
3.4.6	Computational molecules	118
3.4.7	Irregular mesh structures	121
3.5	Test problems	122
4	Standard iterative methods	129
4.1	General theory of iterative methods	129
4.1.1	Stopping criteria	133
4.1.2	Starting vectors	134
4.2	Point iterative methods	135
4.2.1	Basic algorithms	135
4.2.2	Consistent orderings	138
4.2.3	The successive overrelaxation method (SOR)	142
4.2.4	Determining the optimum relaxation parameter	148
4.2.5	Experimental examination of SOR convergence	158
4.2.6	Computational aspects	171
4.3	Line iterative methods	176
4.3.1	1-line algorithms	178
4.3.2	2-line algorithms	189
4.3.3	3-line algorithms	193
4.4	Results of numerical experiments	195
5	Explicit prefactorization methods (AGA)	199
5.1	Matrix notation	200
5.1.1	Basic algorithms	200
5.1.2	AGA algorithms with point modification	208
5.1.3	AGA algorithms with line modification	208
5.1.4	Techniques for accelerating convergence	209
5.2	Implementing prefactorization algorithms in mesh structures	213
5.2.1	Rectangular geometry	213
5.2.2	Triangular geometry	234
5.2.3	Hexagonal geometry	240
5.3	Results of numerical experiments	243
6	Semi-explicit prefactorization with implicit backward sweep	248
6.1	Matrix notation	248
6.1.1	Modified line methods	250
6.2	Implementation in mesh structures	253
6.2.1	Rectangular geometry	253
6.2.2	Triangular geometry	260
6.2.3	Hexagonal geometry	262
6.3	Numerical experiments	265

7	Semi-explicit prefactorization with implicit forward sweep	267
7.1	Implementing OLA methods in mesh structures	268
7.1.1	Rectangular geometry	268
7.1.2	Triangular geometry	285
7.1.3	Hexagonal geometry	291
7.1.4	Successive overrelaxation	294
7.2	Matrix notation	295
7.3	Numerical experiments	302
7.4	Final discussion	304
8	Advances in solving linear control systems	308
8.1	Sylvester equations	308
8.1.1	The SOR-like method	309
8.1.2	Numerical experiments	312
8.1.3	Sylvester equations: conclusion	319
8.2	Continuous-time algebraic Riccati equations	320
8.2.1	The SOR-like method	320
8.2.2	Numerical experiments	321
8.2.3	Concluding remarks	323
A	Numerical experiments for chapter 4	325
A.1	Standard iterative methods: self-adjoint problems	325
A.2	Standard iterative methods: non-self-adjoint problems	351
B	Numerical experiments for chapter 5	375
B.1	AGA algorithms: self-adjoint problems	375
B.2	AGA algorithms: non-self-adjoint problems	455
C	Numerical experiments for chapter 6	473
	(Semi-explicit prefactorization with implicit backward sweep)	
C.1	Self-adjoint problems	473
C.2	Non-self-adjoint problems	487
D	Numerical experiments for chapter 7	496
	(Semi-explicit prefactorization with implicit forward sweep)	
D.1	Self-adjoint problems	496
D.2	Non-self-adjoint problems	514
	Bibliography	529
	Index	535