Additions to Second Edition Errata and Comments

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We thank Dick Palas and Todd Kemp for their contributions.

Page 48 Figure 1.2.5: On the right, the final matrix should be written $A(BC)$, not $(AB)C$.

Page 71 Exercise 1.3.22 belongs in Section 1.4, as it uses the dot product and orthogonality.

Page 138 Remark: In several places we wrote $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ when we meant $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$. The $\vec{\mathbf{v}}$ in the expression $\left[\mathbf{D}f \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right] \vec{\mathbf{v}}$ does not belong there. The last half of the remark should read:

... to a step of length $\sqrt{5}$ in the direction $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$. To take a step of length 1 in that direction, starting at the origin, we would multiply $\left[\mathbf{D}f \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right]$ by $\begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$, which has length 1, to get a rate of ascent (at time 0) of $19/\sqrt{5} \approx 8.5$. In which direction is the function increasing faster, $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ or $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$?

In the footnote, $36/5 \approx 7.2$ should be $36/5 = 7.2$.

Page 144 Line 4: "by direct computation", not "by direction computation".

Page 172 Theorem 2.1.3: To be consistent with later notation, we should write $[A \,|\, \vec{\mathbf{b}}]$, not $[A\,, \vec{\mathbf{b}}]$ and $[A' \,|\, \vec{\mathbf{b}}']$, not $[A', \vec{\mathbf{b}}']$.

First margin note: $[A \,|\, \tilde{\mathbf{b}}]$, not $[A\tilde{\mathbf{b}}]$

We use the vertical line to avoid confusion with the *product* $A\vec{\mathbf{b}}$. You should not think that $\vec{\mathbf{b}}$ is somehow special as far as row reduction is concerned; the rules of row reduction apply equally to all the columns of $[A \,|\, \vec{\mathbf{b}}]$: the columns of A and the column $\vec{\mathbf{b}}$.

Page 176 Exercise 2.1.5, "in the algorithm for row reduction" should be "in Definition 2.1.1 of row operations".

Pages 178, 179, 181 As for Page 172, to keep notation consistent, various augmented matrices should have vertical lines, not commas, as in $[\widetilde{A} \,|\, \widetilde{\vec{\mathbf{b}}}]$.

Page 195 Definition 2.4.5 was perhaps not clear; we mean *an* vector $\vec{\mathbf{w}}$, not some particular $\vec{\mathbf{w}}$. Here is a rewrite:

Definition 2.4.5 (Linear independence). The vectors $\vec{\mathbf{v}}_1, \dots, \vec{\mathbf{v}}_k \in \mathbb{R}^n$ are linearly independent if every vector in $\mathbb{R}^n$ can be written as a linear combination of $\vec{\mathbf{v}}_1, \dots, \vec{\mathbf{v}}_k$ in at most one way, i.e.:

$$\sum_{i=1}^{k} x_i \vec{\mathbf{v}}_i = \sum_{i=1}^{k} y_i \vec{\mathbf{v}}_i \quad \text{implies} \quad x_1 = y_1,\ x_2 = y_2, \dots, x_k = y_k.$$

Page 243 In Definition 2.7.4 we should have required that $U \subset \mathbb{R}^n$ be open.

Page 454 Exercise 4.5.17, part (a): "Let $M_r(\mathbf{x})$ be the rth smallest ... ", not "Let $M_r(\mathbf{x})$ be the rth largest ... ".

Page 465 Exercise 4.6.2: for $k = 1$, we meant the initial conditions to be $x_1 = .7$ and $x_2 = .5$ (not $x_1 = 17$ and $x_2 = .57$).

Page 496 Last margin note: The sentence "At $\varphi = -\pi/2$ and $\varphi = \pi/2$, $r = 0$" should be deleted.

Page 514 Proposition 4.11.15: We should have specified that a and b are constants.

Page 530 Definition 5.1.3: How do we know that $\det(T^\top T) \geq 0$, so that $\sqrt{\det(T^\top T)}$ makes sense? Here is one justification:

Note that

$$(T^\top T)\vec{\mathbf{v}} \cdot \vec{\mathbf{v}} = (T^\top T\vec{\mathbf{v}})^\top \vec{\mathbf{v}} = T\vec{\mathbf{v}} \cdot T\vec{\mathbf{v}} > 0.$$

Denote by A the $k \times k$ matrix $T^\top T$ and let I be the $k \times k$ identity matrix, set $0 \leq t \leq 1$, and consider the matrix $\big(tA + (1-t)I$, which we can think of as A (when $t = 1$) being transformed to I (when $t = 0$). Now, for $\vec{\mathbf{v}} \neq \mathbf{0}$, we have

$$\big(tA + (1-t)I\big)\vec{\mathbf{v}} \cdot \vec{\mathbf{v}} = t\,\underbrace{A\vec{\mathbf{v}} \cdot \vec{\mathbf{v}}}_{>0} + \underbrace{(1-t)}_{\geq 0}\underbrace{\vec{\mathbf{v}} \cdot \vec{\mathbf{v}}}_{>0} > 0.$$

This implies that, for $0 \leq t \leq 1$, $\ker\big(tA + (1-t)I = \mathbf{0}$ and thus that $\det\big(tA + (1-t)I$ is never 0 when $0 \leq t \leq 1$. Since when $t = 0$, $\det\big(tA + (1-t)I = 1$, and when $t = 1$, $\det\big(tA + (1-t)I = \det A$, it follows that $\det A > 0$.

Page 537 Middle margin note: z-axis, not x-axis, in "you get the equation of the surface obtained by rotating the original curve around the x-axis".

Page 545 Line 2, plural, not singular: "the intersection of the surfaces of equations".

Page 551 Exercise 5.3.2: "Use the result of Exercise 5.3.1 (a)", not "use Equation 5.3.1 ... ".

Page 556 Exercise 5.6: Some subscripts got forgotten, and one superscript is wrong. It should be:

(a) Show that $w'_{n+1}(r) = v_n(r)$.

(b) Show that $v_n(r) = r^n v_n(1)$.

(c) Derive Equation 5.3.49, using $w_{n+1}(1) = \int_0^1 w'_{n+1}(r)\, dr$.

Page 563 Clarification for Example 6.1.8:

The function $W_{\vec{\mathbf{v}}}(\vec{\mathbf{w}}) = \vec{\mathbf{v}} \cdot \vec{\mathbf{w}}$ is a 1-form on $\mathbb{R}^n$ because it is a function of one vector and it is linear as a function of $\vec{\mathbf{w}}$. The requirement that it be antisymmetric is automatically satisfied, since it is a function of only one vector.

Page 582 Proposition 6.3.8: We should have said "Suppose there exists a normal vector field $\vec{\mathbf{n}}$", not "Choose a normal vector field $\vec{\mathbf{n}}$". If no normal vector field $\vec{\mathbf{n}}$ exists, then the manifold is not orientable.

Page 592 We forgot to put a $\triangle$ to mark the end of Example 6.4.3.

Page 596 First margin note, third line: there is an extra colon.

Page 622 Last line of Definition 6.6.10: $\partial_1 P$ should be $\partial_M P$.

Page 626 Exercise 6.6.5: $\vec{\nabla}$ denotes the transpose of the derivative:

$$\vec{\nabla} f(\mathbf{x}) = [\mathbf{D} f(\mathbf{x})]^\top.$$

For a function $f : \mathbb{R}^n \to \mathbb{R}$, $\vec{\nabla} f(\mathbf{x})$ is a vector whereas $[\mathbf{D} f(\mathbf{x})]$ is a line matrix.

Note that $\vec{\nabla} f(\mathbf{x}_0)$ is orthogonal at $\mathbf{x}_0$ to the manifold X of equation $f(\mathbf{x}) = 0$: since $T_{\mathbf{x}_0} X = \ker[\mathbf{D} f(\mathbf{x}_0)]$, if $\vec{\mathbf{v}} \in T_{\mathbf{x}_0} X$, then

$$\vec{\nabla} f(\mathbf{x}_0) \cdot \vec{\mathbf{v}} = [\mathbf{D} f(\mathbf{x}_0)] \vec{\mathbf{v}} = 0.$$

Page 629 Last line of Remark 6.7.2: "in higher dimensions", not "to higher dimensions".

Page 635 The formulas for the gradient and the divergence work in any $\mathbb{R}^n$, but there is no obvious generalization of the curl, other than the exterior derivative.

Page 642 We omitted part (c) of Exercise 6.8.10:

(c) Compute it again, directly from the definition of the exterior derivative.

Page 651 Theorem 6.10.2: The reference to Definition 6.6.13 should be to Definition 6.6.10.

Page 657 Exercise 6.10.8 is wrong as written; indeed, it contradicts Exercise 6.10.7. The vector fields should be $\begin{bmatrix} xy^2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ -x^2 y \end{bmatrix}$.